Ejercicios $\mathbb{R}/\mathbb{C}$

1) i) $x^2+x+1 \geq 0$

$x = \frac{-1 \pm \sqrt{-3}}{2} = \nexists \in \mathbb{R}$

	$(-\infty,\infty)$
x^2+x+1	✓

$S=(-\infty,\infty)$

ii) $x<0 \wedge x^2+x-6<0$

$(-\infty,0)$; $x = \frac{-1 \pm \sqrt{25}}{2}$ → 2, -3

	$-\infty,-3$	$-3,2$	$2,\infty$
x^2+x-6	X	✓	X

; $(-3,2)$

$S=(-\infty,0)\cap(-3,2) = (-3,0)$

iii) $\sqrt{\frac{x^2+1}{x^2-1}} \in \mathbb{R} \Rightarrow \frac{x^2+1}{x^2-1} > 0 \wedge x^2-1 \neq 0 \Rightarrow x \neq 1$

$(x^2+1<0 \wedge x^2-1<0) \vee (x^2+1>0 \wedge x^2-1>0)$

$x^2<-1$ $\emptyset$

$x^2>-1$ $(-\infty,\infty)$

$x^2-1>0$; $x^2>1$ → $x>1$, $x<-1$ $\quad (-\infty,-1)\cup(1,\infty)$

$\boxed{(-\infty,-1)\cup(1,\infty) = S}$

iv) $\frac{2x+1}{x+2}<1$; $\frac{2x+1}{x+2}-1<0$; $\frac{2x+1-x-2}{(x+2)}<0$; $\frac{x-1}{x+2}<0$

$(x-1<0 \wedge x+2>0) \vee (x-1>0 \wedge x+2<0)$;

$S=((-\infty,1)\cap(-2,\infty)) \cup ((1,\infty)\cap(-\infty,-2)) = (-2,1)\cup\emptyset = (-2,1)$

v) $(2x+1)^6(x-1)\geq 0 \rightsquigarrow (2x+1)^6(x-1)=0$; $(x-1)=0 \vee (2x+1)^6=0$;
$x=1 \vee 2x+1=0$; $x=-1/2$

	$(-\infty,-1/2)$	$-1/2$	$(-1/2,1)$	1	$(1,\infty)$
$(2x+1)^6(x-1)\geq 0$	X <0	✓ $=0$	X <0	✓ $=0$	✓

$S=\{-1/2\}\cup[1,\infty)$

vi) $x^2+1=0$; $x^2=-1$ $\quad S=\emptyset$

vii) $x^2+x<2$; $x^2+x-2<0$; $x^2+x-2=0 \Rightarrow x=\frac{-1\pm\sqrt{9}}{2}$ → 1, -2

	$(-\infty,-2)$	$(-2,1)$	$(1,\infty)$
$x^2+x<2$	X	✓	X

$S=(-2,1)$

viii) $x < x^2-12 < 4x \Leftrightarrow x < x^2-12 \;\wedge\; x^2-12 < 4x \rightsquigarrow x^2-4x-12<0$

$=0 \Rightarrow x = \frac{4\pm\sqrt{16+48}}{2} = \frac{4\pm 8}{2} \to 6, -2$

$-x^2+x+12<0$

$=0$

$x = \frac{-1\pm 7}{-2} \to -3, 4$

$((-\infty,-3)\cup(4,\infty)) \cap (-2,6) = \boxed{(4,6) = S}$

	$-\infty,-2$	$-2,6$	$6,\infty$
$x^2-4x-12<0$	X	✓	X

	$(-\infty,-3)$	$(-3,4)$	$(4,\infty)$
$0>-x^2+x+12$	✓	X	✓

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ix) $\frac{x-2}{x+3}<0$, $(x-2<0 \wedge x+3>0) \vee (x-2>0 \wedge x+3<0)$

$S = ((-\infty,2)\cap(-3,\infty)) \cup ((2,\infty)\cap(-\infty,-3)) = (-3,2)\cup\emptyset = (-3,2)$

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x) $\frac{x^2-4}{x-1}\geq 0$, $x-1\neq 0$; $\boxed{x\neq 1}$ → solución fuera de los reales

$x^2-4=0$; $x^2=4$; $\boxed{x=2, x=-2}$ → soluciones $\frac{x^2-4}{x-1}=0$

$S=[-2,1)\cup[2,\infty)$

$(x^2-4<0 \wedge x-1<0) \vee (x^2-4>0 \wedge x-1>0)$

$\otimes>0 \Rightarrow ((-2,2)\cap(-\infty,1)) \cup [((-\infty,-2)\cup(2,\infty))\cap(1,\infty)] = (-2,1)\cup(2,\infty)$

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xi) $(x^2+1)(x^2+4)(x^3-1)=0 \Rightarrow x^2+1=0 \vee x^2+4=0 \vee x^3-1=0$

$\emptyset \cup \emptyset \cup x^3=1$; $x=\sqrt[3]{1}=1$

$S=\{1\}$

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xii) $x^2-2x+1\leq 0$

$=0$

$x = \frac{+2\pm\sqrt{\cancel{4-4}}}{2}=1$

	$(-\infty,1)$	1	$(1,\infty)$
$x^2-2x+1=0$	X	✓	X

$S=\{1\}$

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② $a\in\mathbb{R}$, Demuestra:

a) $|x|\leq a \Leftrightarrow -a\leq x\leq a$, $\forall x\in\mathbb{R}$

$|x|\leq a \Rightarrow$ (def |·|) $\begin{cases} x>0 \rightsquigarrow x\leq a; \\ x<0 \rightsquigarrow -x\leq a \Leftrightarrow x\geq -a \end{cases}$ $\Rightarrow -a\leq x\leq a$ ■

$-a\leq x\leq a \Rightarrow \left\{ \begin{array}{l} -a\leq x \;\wedge\; x\leq a \\ \Downarrow \\ a\geq -x \;\wedge\; x\leq a \end{array} \right\}$

$\Rightarrow a\geq -x$, $a\geq x$, si a^+ $\begin{cases} -x>0 \\ 0<x<a \end{cases}$ $\begin{cases} -x, \text{ si } x<0 \\ x, \text{ si } 0<x<a \end{cases}$ $\Rightarrow |x|\leq a$

b) $|x|>a \Leftrightarrow x>a \vee x<-a$

$x>a \vee x<-a \Rightarrow -x>a$

def |·|: $x>0 \Rightarrow x>a$ ■ qed

$x<0 \Rightarrow -x>a \Rightarrow x<-a$

si a^+, $-x$ es x, para $x>0$; x es x, para $0<x<a$ $\Rightarrow$ def $|x|>a$

c) $x^2\leq a \Leftrightarrow -\sqrt{a}\leq x\leq\sqrt{a}$; $x^2\leq a \Leftrightarrow -a\leq x^2\leq a$ $\begin{cases} x^2\geq -a \;\; \emptyset \\ x^2\leq a \Rightarrow \begin{cases} x\leq\sqrt{a} \\ x\geq -\sqrt{a} \end{cases} \end{cases}$ $\Rightarrow \sqrt{a}\leq x\leq\sqrt{a}$

③ i) $|x|\leq 2$; $-2\leq x\leq 2 \Rightarrow S=(-\infty,2]\cap[-2,\infty)=[-2,2]$ ✓

ii) $|x-2|\geq 1$; $x-2\geq 1 \vee x-2\leq -1 \Rightarrow S=[3,\infty)\cup(-\infty,1]$ ✓

iii) $|x|>4$, $x>4 \vee x<-4 \Rightarrow (-\infty,-4)\cup(4,\infty)=S$ ✓

iv) $|x^3-2x+1|\geq 0 \Rightarrow x^3-2x+1\geq 0 \vee x^3-2x+1\leq 0 \Rightarrow S=(-\infty,\infty)$ ✓

se cumple siempre

v) $|x^2-1|\leq 5 \Rightarrow -5\leq x^2-1\leq 5 \Rightarrow -4\leq x^2 \wedge x^2\leq 6 \Rightarrow -4\leq x^2 \wedge |x|\leq\sqrt{6}$

$-4\leq x^2 \Downarrow (-\infty,\infty)$

$|x|\leq\sqrt{6} \Rightarrow -\sqrt{6}\leq x\leq\sqrt{6} \Rightarrow S=(-\infty,\sqrt{6}]\cap[-\sqrt{6},\infty)=$

$=[-\sqrt{6},\sqrt{6}]$ ✓

vi) $1<|x-2|\leq 3 \Rightarrow \underset{ⓐ}{1<|x-2|} \wedge \underset{ⓑ}{|x-2|\leq 3}$

ⓐ $\begin{cases} x-2>1 \vee x-2<-1 \\ (3,\infty)\cup(-\infty,1) = \underline{(-\infty,1)\cup(3,\infty)} \end{cases}$

ⓑ $\begin{cases} x-2\leq 3 \wedge x-2\geq -3 \\ (-\infty,5]\cap[-1,\infty)=\underline{[-1,5]} \end{cases}$

ⓐ $\wedge$ ⓑ $\Leftrightarrow [(-\infty,1)\cup(3,\infty)]\cap[-1,5]=[-1,1)\cup(3,5]=S$ ✓

vii) $|x+2||x-2|>4 \Rightarrow \underset{ⓐ}{x+2>\frac{4}{|x-2|}} \vee \underset{ⓑ}{x+2<\frac{-4}{|x-2|}}$; ⓐ $\left\{ \left(\underset{ⓒ}{|x-2|>\frac{4}{x+2}} \wedge \underset{ⓓ}{x+2>0}\right) \vee \left(\underset{ⓔ}{|x-2|<\frac{4}{x+2}} \wedge \underset{ⓕ}{x+2<0}\right)\right.$

$(-\infty,-2)$

ⓒ $\begin{cases} x-2>\frac{4}{x+2} ; x^2>8 ; |x|>\sqrt{8} ; (x>\sqrt{8}) ; (x<-\sqrt{8}) ; (-\infty,-\sqrt{8})\cup(\sqrt{8},\infty) \\ x-2<\frac{-4}{x+2} ; x^2-4<-4 ; x^2<0 \ (\emptyset) \end{cases}$

ⓒ $\cap$ ⓓ $\Rightarrow (\sqrt{8},\infty)$

ⓔ ⓖ $\begin{cases} x-2<\frac{4}{x+2} ; x^2-4<4 ; |x|<\sqrt{8} ; \boxed{(-\sqrt{8},\sqrt{8})} \Rightarrow \emptyset \\ \quad x<\sqrt{8} \wedge x>-\sqrt{8} \\ x-2>\frac{-4}{x+2} ; x^2-4<-4 ; x^2<0 \ \emptyset \end{cases}$

a el otro lado ⓑ sale $(-\infty,-\sqrt{8})$, luego ⓐ ⓑ $\Rightarrow S=(-\infty,-\sqrt{8})\cup(\sqrt{8},\infty)$

viii) $|x-5|<|x-1|$; $\underline{-|x-1|<x-5<|x-1|}$
ⓐ $\wedge$ ⓑ

ⓨ ⓐ $\left\{ -|x-1|<x-5 \Leftrightarrow |x-1|>-x+5 \Rightarrow x-1>-x+5 \vee x-1<x-5 \Rightarrow 2x>6 \vee \cancel{-1<-5} \right.$

$x>6/2=3 \quad (3,\infty)=ⓐ$

ⓑ $\left\{ x-5<|x-1| \Leftrightarrow |x-1|>x-5 \Rightarrow x-1>x-5 \vee x-1<-x+5 \Rightarrow -1>-5 \vee 2x<6 ; x<3 \right.$

$(-\infty,\infty) \cup (-\infty,3)$

ⓑ $=(-\infty,\infty)$

ⓐ $\cap$ ⓑ $=(-\infty,\infty)\cap(3,\infty)=S=(3,\infty)$ ✓

ix) $|x-1|<|x| \rightsquigarrow x>|x-1| \vee x<-|x-1|$; $-x<x-1<x \vee -x>|x-1| \Rightarrow x<x-1<-x$

$-x<x-1$; $-2x<-1$; $x>\frac{1}{2}$ $(\frac{1}{2},\infty)$

y $\rightarrow (\frac{1}{2},\infty)$

$x-1<x$; $0<1$ $(-\infty,\infty)$

$\left\{ ① \begin{cases} x<x-1 ; 0<-1 \ \emptyset \\ ② \\ x-1<-x \end{cases} \right.$ y $\emptyset \Big\} \Rightarrow S=(\frac{1}{2},\infty)$

(3b) ⓐ ^ ⓑ

x) $|x| + |x+1| < 2$; $|x| < 2 - |x+1|$; $-2+|x+1| < x < 2-|x+1|$

$-2+|x+1| < x$; $|x+1| < x+2$; $-x-2 < x+1 < x+2$; $\begin{cases} -x-2 < x+1 ;\ -2x < 3 ;\ x > -\frac{3}{2} \Rightarrow (-\frac{3}{2}, \infty) \\ (y) \\ x+1 < x+2 ;\ 1 < 2 \quad (-\infty, \infty) \end{cases}$ $(-\frac{3}{2}, \infty)$

$a \wedge b \Rightarrow (-\frac{3}{2}, \infty) \cap (-\infty, \frac{1}{2}) = (-\frac{3}{2}, \frac{1}{2}) = S$

$x < 2-|x+1|$; $-x+2 > |x+1|$; $x-2 < x+1 < -x+2$; $\begin{cases} x-2 < x+1 ;\ -2 < 1 \quad (-\infty, \infty) \\ (y) \\ x+1 < -x+2 ;\ 2x < 1 ;\ x < \frac{1}{2} \end{cases}$ $(-\infty, \frac{1}{2})$

(4)

i) $(6-5i)(6+5i) = (36+25, 30+(-30)) = 61$

ii) $(1-i)(1+2i)(1-3i) = (1+2, 2-1)(1,-3) = (3+3, -9+1) = (6,-8) = 6-8i$

iii) $\frac{1}{2-i} \cdot \frac{2+i}{2+i} = \frac{2+i}{5} = \frac{2}{5} + \frac{1}{5}i$

iv) $\frac{7-4i}{3+2i} \cdot \frac{3-2i}{3-2i} = \frac{(21-8, 14+12)}{13} = \frac{13}{13} + \frac{26}{13}i = 1+2i$

$i = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

	0	30	45	60	90
	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
tg	0	$\sqrt{3}/3$	1	$\sqrt{3}$	$\nexists$

v) $(2-3i)^2 + (i+5)^2 = (2-3i)(2-3i) + (i+5)(i+5) = (-5-12i) + (24, 10) = 19-2i$

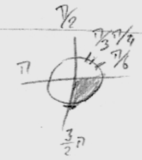
vi) $i^3(1+i)^2 - (2i-1) = -i(1+i)(1+i) - (2i-1) = -i(0+2i) - (2i-1) = 2-(2i-1) = 3+2i$

vii) $\frac{i(7+3i)}{3-4i} \cdot \frac{3+4i}{3+4i} = \frac{(-3,7)(3,4)}{25} = \frac{(-37, 9)}{25} = \frac{-37}{25} + \frac{9}{25}i$

viii) $\frac{(1-i)^3(\sqrt{3}+i)}{(1-\sqrt{3}i)} =$

$= \frac{(1-i)^3(\sqrt{3}+i)}{(1-\sqrt{3}i)} \cdot \frac{(1+\sqrt{3}i)}{1+\sqrt{3}i} = \frac{(1-i)^3(\sqrt{3}-\sqrt{3}, 4)}{4} = \frac{(1-i)^3(0+4i)}{4} = \frac{(-2,-2)(0,4)}{4} = \frac{(8,-8)}{4} = 2-2i$

$(1,-1)(1,-1) = (0,-2)(1,-1) = (-2,-2)$



(5)

i) $1(\cos\pi + i\sin\pi) = 1_\pi = -1$

iii) $1-\sqrt{3}i = 2_{\frac{5\pi}{3}} = 2(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3})$

ii) $1+i = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) = \sqrt{2}_{\pi/4}$

iv) $-i = 1_{3\pi/2} = 1(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2})$

$\frac{9}{6}\pi + \frac{\pi}{6}$ $\frac{10\pi}{6}$ = $\frac{5}{3}\pi$

v) $(\sqrt{3}+i) = 2_{\pi/6} = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$

$\arctg\frac{1}{0} \nexists \Rightarrow \theta(\frac{\pi}{2} \text{ o } \frac{3\pi}{2})$

$\arctg\frac{1}{\sqrt{3}} = \arctg\frac{\sqrt{3}}{3} = (\frac{\pi}{6}?)$ ✓

(6)

i) $2e^{i\pi} \Rightarrow \begin{array}{l} a = 2\cos\pi = -2 \\ b = 2\sin\pi = 0 \end{array} \Rightarrow 2e^{i\pi} = (-2, 0)$

ii) $e^{i\pi/2} \Rightarrow a = 1\cos-\frac{\pi}{2} = 0 \Rightarrow e^{-i\pi/2} = (0,-1)$

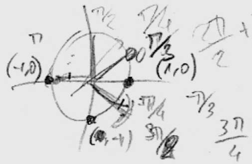
$b = 1\sin-\frac{\pi}{2} = -1\cdot(-\sin\frac{\pi}{2}) = 1\cdot(-1)$

iii) $\sqrt{5} \Rightarrow (\sqrt{5}, 0)$

iv) $3e^{i\pi/3} \Rightarrow \begin{array}{l} a = 3\cdot\cos\frac{\pi}{3} = \frac{3}{2} \\ b = 3\cdot\sin\frac{\pi}{3} = \frac{3\sqrt{2}}{2} \end{array} \Rightarrow 3e^{i\pi/2} = (\frac{3}{2}, \frac{3\sqrt{2}}{2})$

vi) $e^{i\pi/6} \Rightarrow \begin{array}{l} a = 1\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \\ b = 1\sin\frac{\pi}{6} = \frac{1}{2} \end{array} \Rightarrow e^{i\pi/6} = (\frac{\sqrt{3}}{2}, \frac{1}{2})$

v) $\sqrt{2}e^{-i\pi/4} \Rightarrow \begin{array}{l} a = \sqrt{2}\cos\frac{-\pi}{4} = \sqrt{2}\cdot\cos\frac{\pi}{4} = \frac{\sqrt{2}\cdot\sqrt{2}}{2} = 1 \\ b = \sqrt{2}\cdot\sin-\frac{\pi}{4} = \sqrt{2}\cdot(-\sin\frac{\pi}{4}) = \frac{\sqrt{2}(-\sqrt{2})}{2} = -1 \end{array} \Rightarrow \sqrt{2}e^{-i\pi/4} = (1,-1)$



$\frac{2\pi}{2}+\frac{\pi}{2}=\frac{3\pi}{2}$ $\quad \frac{6}{4}\pi+\frac{\pi}{4}=\frac{7\pi}{4}$ $\quad \frac{3\pi}{4}+\frac{\pi}{4}=\frac{4\pi}{4}=\pi$ $\quad \frac{6\pi}{3}-\frac{\pi}{3}=\frac{5\pi}{3}$

(7) i) $\sqrt[5]{1} \Rightarrow \theta=\arctan\frac{0}{1}=0 \Rightarrow \sqrt[5]{1}_{0+\frac{2k\pi}{5}}, \ k=0,1,\dots,4$

ii) $\sqrt[4]{-1} \Rightarrow \theta=\arctan\frac{0}{-1}=\pi \Rightarrow \sqrt[4]{1}_{\pi} \rightsquigarrow = 1_{\frac{\pi}{4}+\frac{k\pi}{2}}, \ k=0,1,2,3$

iii) $\sqrt[3]{1-i} \Rightarrow \theta=\arctan\frac{-1}{1}=\frac{7\pi}{4} \Rightarrow \sqrt[3]{\sqrt{2}}_{\frac{7\pi}{4}} \rightsquigarrow = \sqrt[6]{2}_{\frac{7\pi}{12}+\frac{2k\pi}{3}}, \ k=0,1,2$

iv) $\sqrt[4]{i/2} = \sqrt[4]{0+\frac{1}{2}i} \Rightarrow \theta=\underset{\nexists}{\arctan\frac{1/2}{0}} = \begin{cases}\pi/2\\ 3\pi/4\end{cases} \Rightarrow \sqrt[4]{1/4}_{\pi/2} = \left(\frac{1}{\sqrt[8]{4}}\right)_{\frac{\pi}{8}+\frac{2k\pi}{4}} = \left(\frac{1}{\sqrt[4]{2}}\right)_{\frac{\pi}{8}+\frac{k\pi}{2}}, \ k=0,1,2,3$

(8) i) $(-1+i)^3 = (-1,1)(-1,1)(-1,1) = (\overset{0}{1-1}, \overset{-2}{-1-1})(-1,1) = (2,2) = 2+2i$

ii) $(1-\sqrt{3}i)^4 = (2_{\frac{5\pi}{3}})^4 = 16_{\frac{20\pi}{3}} = 16\cdot\cos\frac{20\pi}{3}+i\sin\frac{20}{3}\pi = 16e^{i\frac{20\pi}{3}}$

$-\arctan(\sqrt{3}) =$

12, 12, 24, 12, 144 $\quad$ 60 $\quad$ 119

iii) $(5-12i)^2 = (5,-12)(5,-12) = (25-144, -60-60) = 119-120i$

(9) $i^0=1$, $i^1=i$, $i^2=-1$, $i^3=-i$, $i^4=1$, ⋮

$i^{4k}=1$; $i^{4k+1}=i$; $i^{4k+2}=-1$; $i^{4k+3}=-i$, $\forall k\geq 0$

(10) i) $\cos a \overset{?}{=} \frac{e^{ia}+e^{-ia}}{2} \rightsquigarrow \frac{e^{ia}+e^{-ia}}{2} = \frac{\cos a+i\sin a+\cos(-a)+i\sin(-a)}{2} =$

$= \frac{\cos a+\cancel{i\sin a}+\cos a-\cancel{i\sin a}}{2} = \frac{\cancel{2}\cos a}{\cancel{2}} = \cos a$ $\boxed{■\ \text{qed}}$

ii) $\sin a = \frac{e^{ia}-e^{-ia}}{2i} = \frac{\cos a+i\sin a-(\cos a+i\sin a)}{2i} = \frac{2i\sin a}{2i} = \sin a$ $\boxed{■\ \text{qed}}$

(11) $x^2+4x+29=0$

i) $x=\frac{-4\pm\sqrt{-100}}{2} = \frac{-4\pm\sqrt{-1}\sqrt{100}}{2} = \frac{-4\pm 10i}{2} \to -2+5i, \ -2-5i$

$\pi-\frac{\pi}{3} = \frac{3\pi}{3}-\frac{\pi}{3} = \frac{2\pi}{3}$

$\theta=\arctan\frac{\sqrt{3}}{-1} = -\arctan\sqrt{3} = -\frac{\pi}{3} = \frac{2\pi}{3}$

ii) $r^4+r^2+1=0 \to r^2=t, \ r=\sqrt{t}$

$t^2+t+1=0$

$t=\frac{-1\pm\sqrt{-3}}{2} = \frac{-1\pm\sqrt{-1}\sqrt{3}}{2} = \frac{-1\pm\sqrt{3}i}{2} \to -\frac{1}{2}+\frac{\sqrt{3}}{2}i, \ -\frac{1}{2}-\frac{\sqrt{3}}{2}i$

$r=\sqrt{-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i} = \sqrt{\frac{1}{4}+\frac{3}{4}}_{\frac{\theta}{2}+\frac{2k\pi}{2}}, k=0,1 = \sqrt{1}_{\frac{2\pi}{3\cdot 2}+k\pi} \ (k=0,1) = \begin{cases} k=0: 1_{\pi/3} \\ k=1: 1_{\frac{4\pi}{3}}\end{cases}$

$\theta=\arctan\frac{-\sqrt{3}}{-1} = \pi/3$ $= \sqrt{1}_{\frac{\pi}{6}+k\pi} = \begin{cases} k=0: 1_{\frac{\pi}{6}} \\ 1_{\frac{\pi}{6}+\pi} = 1_{\frac{7\pi}{6}}\end{cases}$

iii) $t^3+t^2-t-1=0 \Rightarrow (t-1)(t+1)(t+1)=0 \Rightarrow \begin{cases} t=1 \\ t=-1\end{cases}$

	1	1	-1	-1
1		1	2	1
	1	2	1	0

	1	2	1
-1		-1	-1
	1	1	0
-1		-1	
	1	0	

(12) $ax^2+bx+c=0 \Rightarrow x = 3 \pm \sqrt{5}i$

i) $3 \pm \sqrt{5}i = \dfrac{6 \pm 2\sqrt{5}i}{2} = \dfrac{6 \pm \sqrt{-20}}{2}$
$\begin{cases} a=1 \\ b=-6 \\ -36+4c=-20 \Rightarrow c = \dfrac{-56}{-4} = 14 \quad c=14 \end{cases} \Rightarrow x^2-6x+14=0$ ✓

ii) $-3 \pm i = \dfrac{-6 \pm 2i}{2} = \dfrac{-6 \pm \sqrt{-4}}{2}$
$\begin{cases} a=1 \\ b=6 \\ c \Rightarrow 36-4c=-4 \Leftrightarrow c = \dfrac{40}{4} = 10 \end{cases}$ $x^2+6x+10=0$ ✓

iii) $2 \pm \sqrt{3}i \sim \dfrac{4 \pm 2\sqrt{3}i}{2} = \dfrac{4 \pm \sqrt{-12}}{2}$
$\begin{cases} a=1 \\ b=-4 \\ 16-4c=-12 \Leftrightarrow c = \dfrac{-28}{-4} = 7 \end{cases}$ $x^2-4x+7=0$ ✓

iv) $-1 \pm 2i \sim \dfrac{-2 \pm 4i}{2} = \dfrac{-2 \pm \sqrt{-16}}{2}$
$\begin{cases} a=1 \\ b=2 \\ c \Rightarrow 4-4c=-16 \Rightarrow c = \dfrac{20}{4} = 5 \end{cases}$ $x^2+2x+5=0$ ✓

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(13) A, w

Datos
$\begin{cases} |A| \cdot |w| = 9 \\ \dfrac{|A|}{|w|} = 1 \end{cases}$ $\begin{cases} \arg(A \cdot w) = 0 \\ \arg\left(\dfrac{A}{w}\right) = \dfrac{\pi}{2} \end{cases}$

Sabemos que
$A \cdot w = |A||w|_{\theta+\mu}$
$\dfrac{A}{w} = \dfrac{|A|}{|w|}_{\theta-\mu}$ $\Rightarrow$ *

$|A||w| = 9 \Rightarrow |A| = \dfrac{9}{|w|}$
$\dfrac{|A|}{|w|} = 1$ $\quad \dfrac{9}{|w||w|} = 1 \Rightarrow |w||w| = 9$
$|w| = \sqrt{9}$
$|A| = \dfrac{9}{\sqrt{9}} \dfrac{\sqrt{9}}{\sqrt{9}} = \sqrt{9}$

$|w| = |A| = \sqrt{9}$

* $\Rightarrow \begin{cases} \theta+\mu=0 \Rightarrow \theta=-\mu \quad \Rightarrow \mu = \dfrac{-\pi}{4} = \dfrac{8\pi}{4} - \dfrac{\pi}{4} = \dfrac{7\pi}{4} \\ \theta-\mu = \dfrac{\pi}{2} \Rightarrow \theta+\theta = \dfrac{\pi}{2} \Rightarrow 2\theta = \dfrac{\pi}{2} \Leftrightarrow \theta = \dfrac{\pi}{4} \end{cases}$

solución
$A = 3_{\pi/4}$
$w = 3_{7\pi/4}$ ✓

(14) $A_1, A_2 \in \mathbb{C}$

$A_1^2 + A_2^2 = 3$

$\frac{A_1}{A_2} = bi$ (imaginario)

$\left|\frac{A_1}{A_2}\right| = 2$

$\Big\}\ \frac{A_1}{A_2} = \pm 2i$

Suponemos caso A $\Big\{$ $A_1 = 2iA_2 \Rightarrow (2iA_2)^2 + A_2^2 = 3 \Rightarrow -4A_2^2 + A_2^2 = 3$

$\Rightarrow -3A_2^2 = 3 \Rightarrow A_2^2 = -1 \Rightarrow \boxed{A_2 = \pm i}$

$A_2 = \pm i \Rightarrow \Big\{$ $A_1 = 2i \cdot i = 2i^2 = -2 = A_1$

$A_1 = 2i \cdot -i = -2i^2 = 2 = A_1$

caso B $\Big\{$ $A_1 = -2iA_2 \Rightarrow \dots \Rightarrow A_2 = \pm i$ $\Big\{$ $A_2 = i \Rightarrow A_1 = 2$; $A_2 = -i \Rightarrow A_1 = -2$

soluciones $\boxed{A_2 = \pm 1,\ A_1 = \mp 2 \quad \text{ó} \quad A_2 = \pm i,\ A_1 = \pm 2}$

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(15) $A^3 - (1+3i)A^2 + (-2+i)A = 0$

$A\left(A^2 - (1+3i)A + (-2+1)\right) = 0$ $\Big\{$ $A = 0$; $A^2 - (1+3i)A + (-2+1) = 0$

$A = \frac{1+3i \pm \sqrt{-8+6i+8-4i}}{2} = \frac{1+3i \pm \sqrt{2i}}{2} = \frac{1+3i \pm 1+i}{2} =$

$= \frac{2+4i}{2} = 1+2i$

$\frac{2i}{2} = i$

SOLUCIONES:

$A = 0$

$A = 1+2i$

$A = i$

Cálculos

$(1+3)(1+3) = (-8+6i)$

$\sqrt{2i} = |2i|^{\frac{1}{2}}$

$\frac{\theta + 2\pi k}{2},\ k = 0,1$

$= \sqrt{2}_{\pi/4} = \frac{\sqrt{2}\cdot\sqrt{2}}{2} + \frac{\sqrt{2}\sqrt{2}}{2}i = 1+i$

$= \sqrt{2}_{5\pi/4} = -\frac{\sqrt{2}\cdot\sqrt{2}}{2} - \frac{\sqrt{2}\sqrt{2}}{2}i = -1-i$

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(16) $A_1 = a + bi$

$A_2 = c + di$

$A_3 = e + fi$

$\overbrace{(a^2-b^2,\ 2ab)}^{A_1^2}\overbrace{(a,b)}^{A_1} = (\dots,\ \underbrace{a^2b - b^3 + 2a^2b}_{-b^3+3a^2b}) = 0 \in \mathbb{R} \Rightarrow$ $a^2b - b^3 + 2a^2b = 0$

$c^2d - d^3 + 2c^2d = 0$

$\boxed{f = 0}$, $\boxed{e \text{ conocido}}$

$a^2 + b^2 = c^2 + d^2$ $\Big\}$ $(b = -d) \Rightarrow a^2 + (-d)^2 = c^2 + d^2 \Leftrightarrow |a| = |c| \Leftrightarrow a = \pm c$

ya que $a+c+e=0$, $a+c=-e$, $a+c \neq 0$; $a = -c$ → no tendría sentido; luego $a = c$

$a + c + e = 0$; $a + a = -e \Rightarrow 2a = -e \Rightarrow$ $\boxed{a = \frac{-e}{2}}$ $\boxed{c = \frac{-e}{2}}$

$b + d = 0 \Leftrightarrow \boxed{b = -d}$

$\frac{(-e)^2}{4}b - b^3 + 2\frac{(-e)^2}{4}b = 0$; $\frac{e^2}{4}b - b^3 + \frac{e^2}{2}b = 0$; $b\left(\frac{e^2}{4} - b^2 + \frac{e^2}{2}\right) = 0$ $\Big\{$ $b = 0$; $\frac{3e^2}{4} - b^2 = 0$; $\boxed{b = \sqrt{3/4}\,e}$

$\boxed{d = -\sqrt{3/4}\,e}$

(17)

a) $\sqrt{(2a+3)^2+2b^2} < 1 \iff (2a+3)^2 + 2b^2 < 1^2 = 1 \iff \left(a+\frac{3}{2}\right)^2 + b^2 < \frac{1}{2} \Rightarrow$ círculo de $R < \frac{1}{\sqrt{2}}$, centro $\left(\frac{-3}{2}, 0\right)$

b) $\frac{|z-i|}{|z+i|} = 2$; $\frac{\sqrt{a^2+(b-1)^2}}{\sqrt{a^2+(b+1)^2}} = 2$; $\frac{a^2+(b-1)^2}{a^2+(b+1)^2} = 4$; $\frac{a^2+b^2-2b+1}{a^2+b^2+2b+1} = 4$;

$a^2+b^2-2b+1 = 4a^2+4b^2+8b+4 \Rightarrow 3a^2+3b^2+10b+3=0 \iff \underline{a^2+b^2+\frac{10}{3}b+1=0}$

completamos cuadrados

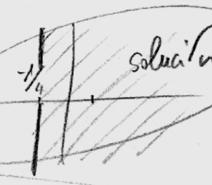
círculo de centro $(0, -5/3)$ y radio $(4/3)$

$\ldots \left(b+\frac{5}{3}\right) = \frac{16}{9} \ldots$

$\Rightarrow a^2 + \left(b+\frac{5}{3}\right)^2 - \frac{16}{9} = 0$; $a^2 + \left(b+\frac{5}{3}\right)^2 = \frac{16}{9}$ ⇝ (R^2)

c) $|2z| \leq |2z+1|$; $\sqrt{4a^2+4b^2} \leq \sqrt{(2a+1)^2+4b^2}$; $\cancel{4a^2} + \cancel{4b^2} \leq \cancel{4a^2} + 4a + 1 + \cancel{4b^2}$;

$0 \leq 4a+1$; $a \geq -1/4$



: semiplano derecho a la recta $x = -1/4$, incluyendo la recta.

d) $Re\left(\frac{2}{z}\right) + Im\left(\frac{4}{\bar{z}}\right) < 1$; $\frac{2a}{a^2+b^2} + \frac{4b}{a^2+b^2} < 1$; $\frac{2a+4b}{\cancel{a^2+b^2}} < a^2+b^2$; $\underline{a^2-2a} + \underline{b^2-4b} > 0$

completamos cuadrados

$(a-1)^2 + (b-2)^2 > 5 \Rightarrow$ circunferencia radio $> \sqrt{5}$, centro $(1,2)$

(R^2)

Problema (1a) EXAMEN FINAL FON. 9-11-2010

$a,b \in \mathbb{R}$, $z \in \mathbb{C}$. $\left\{ z \in \mathbb{C} \mid Im\left(\frac{a+z}{b+z}\right) = Im(z) \right\}$, Para $z = x+yi$

$\frac{(a+x)+yi}{(b+x)+yi} \cdot \frac{(b+x)-yi}{(b+x)-yi} = \frac{-(a+x)y+(b+x)y}{(b+x)^2+y^2} = y \iff \frac{\cancel{y}\left(-(a+x)+(b+x)\right)}{\cancel{y}\left((b+x)^2+y^2\right)} = 1 \iff -a\cancel{-x}+b\cancel{+x} = (b+x)^2+y^2$

circunferencia de radio $\sqrt{-a+b}$ y centro $(-b, 0)$

para $-a+b>0$, $y \neq 0$

si $-a+b<0$, no tiene solución

si $y=0$, todas las soluciones x, $\forall a,b \in \mathbb{R}$ validan el ejercicio, luego

$y=0 \iff$ solución eje OX